Equivalence Principle and Partition of Angular Momenta in the Nucleon

O.V. Teryaev [1]

BLTP,
Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia

Abstract. The manifestation of equivalence principle (EP) in spin-gravity interactions,resulting in the nullification of the corresponding analog of Anomalous Magnetic moment is explored. Its tests in the experiments with atoms and cold neutrons are discussed. The validity of EP separately for quarks and gluons in the nucleon resulting in exact equipartition of momentum and total angular momentum is conjectured. The important role of relocalization (Belinfante) invariance in these and other aspects of nucleon spin structure is stressed.

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Equivalence principle is known to be one of the basic postulates of the modern physics, constituting the cornerstone of General Relativity. Its simplest and well-known "Newtonian" counterpart corresponds to the equality of inertial and gravitational mass and is tested with good accuracy. At the same time, there is another, "post-Newtonian", manifestation of equivalence principle which corresponds to the interaction of *spin with gravity* [1]. It means the absence of gravitational analogs of electric dipole and anomalous magnetic moments. It may be derived as a low energy theorem due to the conservation of momentum and orbital angular momentum. As soon as these conservation laws control (due to Ji sum rules for Generalized Parton Distributions) the partition of momentum and angular momentum between quarks and gluons, the equivalence principle is manifested in this context[2]. The connection is provided by gravitational formfactors, being the matrix elements of Belinfante energy-momentum tensors, and in, turn, to the total angular momenta of partons,

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M]u(p), \tag{1}$$

where $P^\mu = (p^\mu + p^{\mu\prime})/2$, $\Delta^\mu = p^{\mu\prime} - p^\mu$, and $u(p)$ is the nucleon spinor. We dropped here the irrelevant terms of higher order in Δ, as well as containing $g^{\mu\nu}$. The parton momenta and total angular momenta are:

$$P_{q,g} = A_{q,g}(0),$$
$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \ . \tag{2}$$

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[1] teryaev@theor.jinr.ru

Taking into account the conservation of momentum and angular momentum one get

$$\sum_{i=q,G}\int_0^1 dxxH_i(x,\xi,Q^2) = A_q(0)+A_g(0) = 1 \tag{3}$$

$$\sum_{i=q,G}\int_0^1 dxx(H+E)_i(x,\xi,Q^2) = A_q(0)+B_q(0)+A_g(0)+B_g(0) = 1, \tag{4}$$

which is just EP. Note that gravitational analog of dipole moment is absent, as it violates EP as well as CP invariance, while the obvious gravitational analog of anapole moment of the form

$$\bar{u}(p')\gamma^{(\alpha}\gamma_5 p^{\nu)}(\Delta^2 g^{\mu\alpha} - \Delta_\alpha\Delta_\mu)u(p)$$

is allowed. The spin-dipole coupling appears at the level of Hamiltonian entering the unitary transformed Dirac equation [3], although it disappears [4] when the corresponding transformation of dynamical variables is properly considered.

Note that these formfactors describe also the interaction of nucleons with TeV scale extra-dimensional gravity and they should be taken into account when respective gravitational effects in diffractive scattering [5] are considered.

The dedicated test of the nullification of total Anomalous Gravitomagnetic moment (AGM) B was not yet performed. However, there is a recent [2] claim [6] that spin-rotation coupling should be already taken into account when analyzing the data obtained in the precise EDM experiment [7]. Moreover, earlier atomic experiment [8] may be interpreted [9] as a test of EP with a few percent accuracy. Originally this experiment was aimed on the search of gravitational dipole term, but as it violates also CP invariance, the CP conserving EP violating effects should be considered as a dominant ones,

Ultracold neutrons can also be used in interferometer experiments with the rotating spin-flippers [10] and implemented at the existing and developed interferometers at ILL and Tokai [11]. It seems reasonable to have two (rather than one as suggested in [10]) rotating spin-flippers. Their rotation in the same and opposite directions may provide a number of true and false signals. Namely, signals should be absent if they are rotated in the same directions, and it should be twice larger in the case of rotation in opposite direction in comparison yo the case when only one flipper rotates. In total, there are 8 signals which may significantly increase the statistics. Note also that EP tests may be performed in the experiments with polarized electrons and positrons in storage rings [9]

There are also some evidences supporting the conjecture [12] that EP is valid separately for quarks and gluons in the nucleon, which is violated in perturbative QCD but may be restored in full non perturbative (NP)QCD due to the phenomena of confinement and spontaneous chiral symmetry breaking. This (extended) EP means exact equipartition (EEP) of momenta and angular momenta in a nucleon.

The most precise numerical support is coming from the lattice simulations [13] obtained after the conjecture [12]. Another one is coming from the numerical relations between anomalous moments and spin-averaged distributions of valence quarks [14]. It

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[2] I am indebted to J. Ellis for pointing out this reference

is based on the adopted parametrizations of GPD (see also [15]) and the physical reason is nothing else than EEP which holds in this parametrization.

As soon as EEP is related to properties of NP QCD one may expect that it is valid also for other hadrons, in particular for vector mesons. This is supported by QCD sum rules calculations [16] of the anomalous magnetic moment of ρ mesons. The results are close to value $g = 2$ which may be explained as a result of the smallness of the analog of anomalous GPD E, related in turn, to EEP. Note that the gluonic momentum calculated in similar [17] approach is sizable and one cannot attribute EEP merely to absence of gluonic contributions, like in model calculations [15]. Of course, the direct QCD SR calculations of AGM in line with [16] would be very interesting.

The generality of EEP should imply also its validity for the case of hadrons substituted by currents, which would change the matrix elements to 3-point correlators. This may be especially suitable for lattice calculations, when this correlator may be considered as an order parameter for confinement (chiral) phase transition, when EEP may be violated in deconfined phase. If one consider EEP in the case of tensor currents, the respective order parameter looks quite symmetric.

Let us also note that EEP may be supported by the conjecture [18] relating Sivers functions and anomalous magnetic moments. Let us suppose that this relation may be quantified as a proportionality between GPD E and Sivers function. If so, EP directly corresponds to Burkardt [19] sum rule. Furthermore, EEP is a natural counterpart of the recent conjecture [20] about the smallness of gluon Sivers function, supported by COMPASS data [21].

EEP is related to the important property of Relocalization (Belinfante) Invariance (RI) providing the possibility to perform a transformation of the densities of conserved charges and represent the total angular momentum in an "orbital' form with Belinfante symmetrized energy momentum tensor (EMT). Let us stress once more that it is this tensor which describes the coupling to gravity and enters the gravitational formfactors.

The matching of RI with quantum theory happens not to be trivial. The analysis of leading order QCD evolution [22] based on consistent exploration of conservation laws shows that it is RI that leads to EEP due to relation between spin-dependent and spin-independent kernels

$$\int_0^1 dxx\Delta P_{Gq}(x) = \frac{1}{2}\int_0^1 dxxP_{Gq}(x). \tag{5}$$

While RI for classical fields requires their decrease at infinity it puts the constraints for the behaviour of the matrix elements of respective operators at low momentum transfers which are of special interest in In the non-perturbative case, when RI leads [23] to the relation

$$q^2\frac{\partial}{\partial q^\alpha}\langle P|J^\alpha_{5S}|P+q\rangle = (q^\beta\frac{\partial}{\partial q^\beta} - 1)q_\gamma\langle P|J^\gamma_{5S}|P+q\rangle, \tag{6}$$

valid up to the terms linear in q and excluding the possibility of massless pole in the matrix element of singlet axial current. Thus RI provides a complementary view to such important property of NP QCD as $U_A(1)$problem.

Note also that RI provides a guideline for dealing with non-local operators. Indeed, matrix element of the contribution od antisymmetric part of quark EMT to angular momentum should be equal, due to IR, to that of quark spin. This requires that derivative resulting from x factor in angular momentum should act just to matrix element, picking the term linear in Δ, while the singular coefficient is the same delta-function as for local operators. This provides the regular way of deriving the sum rules with no need to use the wave packets like in the detailed analysis of [24].

The resulting sum rules for the longitudinal and transverse [25] polarization are completely similar. Say, the quark spin is described by the different projections of *chiral-even* axial current. The appearance of chiral-odd structures [24] is due to the use of the partonic description in terms of wave functions, for which chiral even and odd structures are related.

To conclude, various experimental checks of equivalence principle in spin-gravity interactions may be extracted from current and future experiments [9]. The generalization of EP in NPQCD is supported by a number of observations and may be related to Belinfante invariance. The possibility of deep roots of EEP in some NPQCD relations to gravity, say, to AdS/CFT correspondence, is very interesting.

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